10.11

Given: $\vec{r}(t) = t^2\,[\mathrm{m/s^2}]\,\hat{\imath} + e^{t\,[1/s]}\,[\mathrm{m}]\,\hat{\jmath}$

$$\therefore\ \vec{v}(t) = \frac{d\vec{r}}{dt} = \boxed{2t\,[\mathrm{m/s^2}]\,\hat{\imath} + e^{t\,[s^{-1}]}\,[\mathrm{m/s}]\,\hat{\jmath}}$$

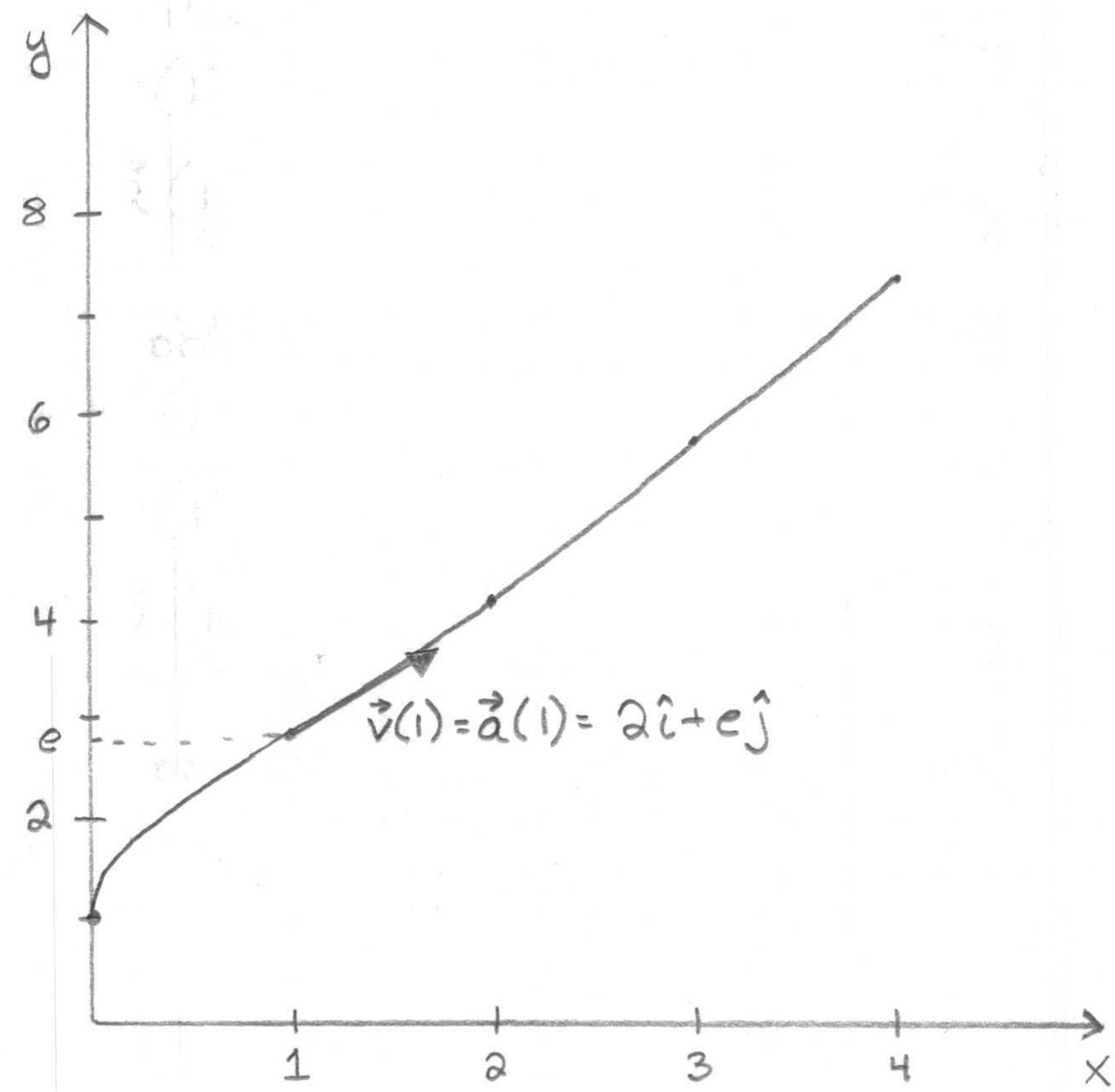
$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \boxed{2\,[\mathrm{m/s^2}]\,\hat{\imath} + e^{t\,[s^{-1}]}\,[\mathrm{m/s^2}]\,\hat{\jmath}}$$

Find $\vec{r}(1) = \hat{\imath} + e\hat{\jmath}\quad[\mathrm{m}]$

$\vec{v}(1) = 2\hat{\imath} + e\hat{\jmath}\quad[\mathrm{m/s}]$

$\vec{a}(1) = 2\hat{\imath} + e\hat{\jmath}\quad[\mathrm{m/s^2}]$

Trajectory:



10.17

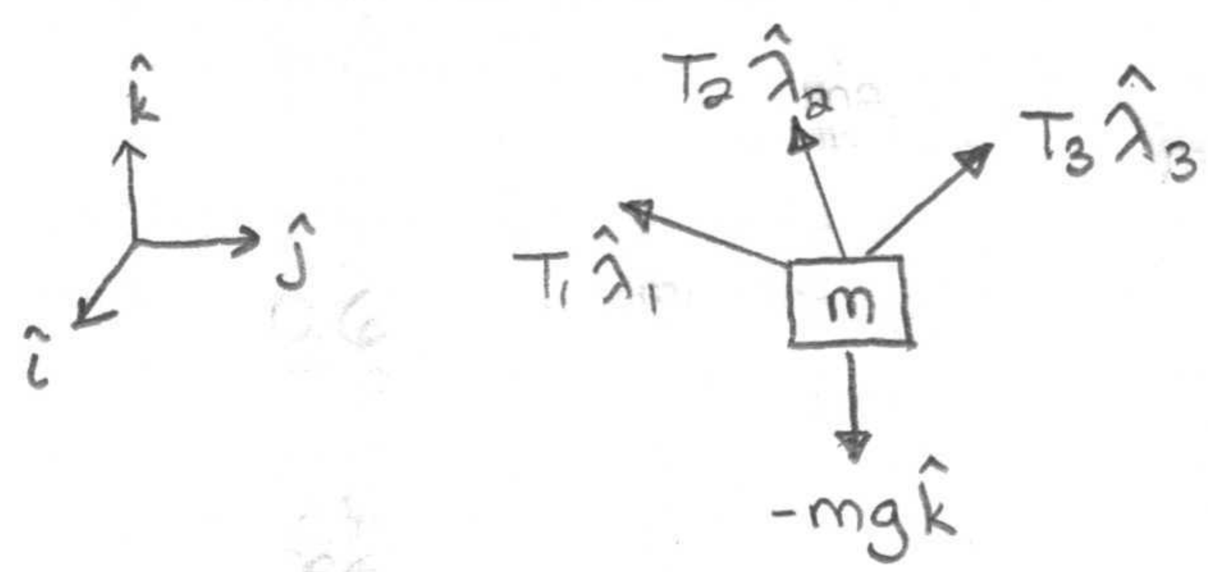
$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ $[m/s^2]$, $m = 3$ kg

See actual Matlab code on next page.

10.22

$\vec{a} = -0.6\hat{i} - 0.2\hat{j} + 2.0\hat{k}$ $[m/s^2]$, $m = 2$ kg

a) Free body diagram:



$\vec{r}_m = 1.0\hat{i} + 1.5\hat{j}$ [m]

$\vec{r}_1 = 2.0\hat{i} + 2.0\hat{k}$ [m]

$\vec{r}_2 = 2.0\hat{k}$ [m]

$\vec{r}_3 = 4.0\hat{j} + 2.0\hat{k}$ [m]

b) $\vec{r}_{m1} = \vec{r}_1 - \vec{r}_m = 1.0\hat{i} - 1.5\hat{j} + 2.0\hat{k}$ $\quad |\vec{r}_{m1}| = \frac{1}{2}\sqrt{29}$

$\vec{r}_{m2} = \vec{r}_2 - \vec{r}_m = -1.0\hat{i} - 1.5\hat{j} + 2.0\hat{k}$ $\quad |\vec{r}_{m2}| = \frac{1}{2}\sqrt{29}$

$\vec{r}_{m3} = \vec{r}_3 - \vec{r}_m = -1.0\hat{i} + 2.5\hat{j} + 2.0\hat{k}$ $\quad |\vec{r}_{m3}| = \frac{3}{2}\sqrt{5}$

$$\sum \vec{F} = m\vec{a} = T_1\hat{\lambda}_1 + T_2\hat{\lambda}_2 + T_3\hat{\lambda}_3 - mg\hat{k}$$

OR $2\,\text{kg}(-0.6\hat{i} - 0.2\hat{j} + 2.0\hat{k}) = T_1\left(\frac{2\sqrt{29}}{29}\right)(\hat{i} - 1.5\hat{j} + 2.0\hat{k})$

$+ T_2\left(\frac{2\sqrt{29}}{29}\right)(-\hat{i} - 1.5\hat{j} + 2.0\hat{k}) + T_3\left(\frac{2\sqrt{5}}{15}\right)(-\hat{i} + 2.5\hat{j} + 2.0\hat{k}) - 19.62\hat{k}$

In component form:

$\hat{i}\,(-1.2) = \hat{i}\,(0.3714T_1 - 0.3714T_2 - 0.2981T_3)$

$\hat{j}\,(-0.4) = \hat{j}\,(-0.5571T_1 - 0.5571T_2 + 0.7454T_3)$

$\hat{k}\,(4 + 19.62) = \hat{k}\,(0.7428T_1 + 0.7428T_2 + 0.5963T_3)$

CONTINUED ON PAGE 4

```
% Problem 10.17 Solution

% Define Constants
m= 3;
a= [1 2 3]';

% Define Positions
rAB= [2 3 5]';
rAC= [-3 4 2]';
rAD= [1 1 1]';
uAB= rAB/norm(rAB); %lambda(AB)
uAC= rAC/norm(rAC); %lambda(AC)
uAD= rAD/norm(rAD); %lambda(AD)

% Formulate lambda matrix
U= [uAB uAC uAD];

% Solve for tensions
T= U\(m*a);
```

```
% Problem 10.22 Solution

% Define Constants
m= 2;
a= [-.6 -.2 2]';
g= [0 0 -9.81]';

% Define Positions
rAB= [1 -1.5 2]';
rAC= [-1 -1.5 2]';
rAD= [-1 2.5 2]';
uAB= rAB/norm(rAB); %lambda(AB)
uAC= rAC/norm(rAC); %lambda(AC)
uAD= rAD/norm(rAD); %lambda(AD)

% Formulate lambda matrix
U= [uAB uAC uAD]

% Solve for tensions
T= U\(m*a-m*g)
```

10.22 continued.

In matrix form, we have:

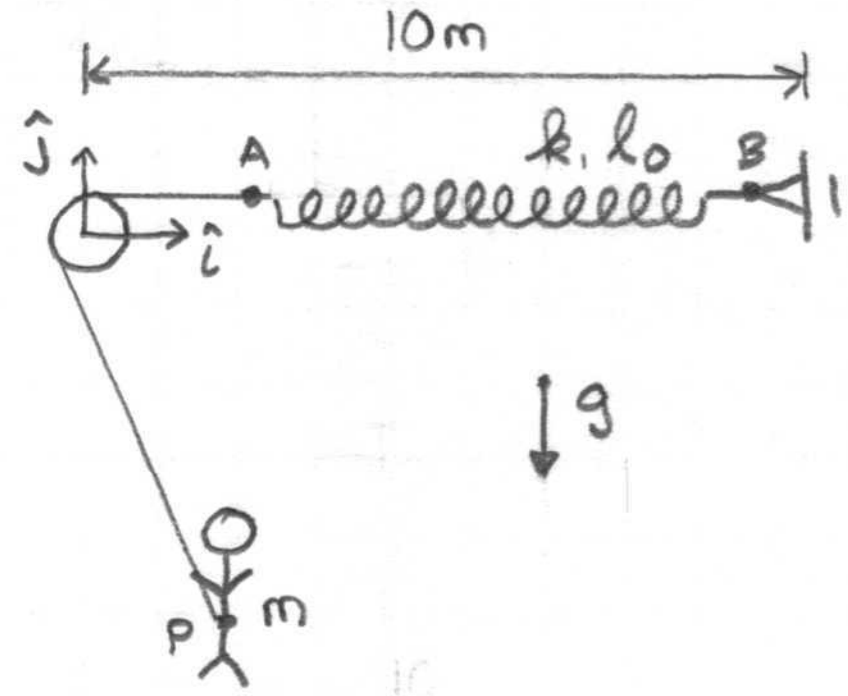
$$\begin{bmatrix} -1.2 \\ -0.4 \\ 23.62 \end{bmatrix} = \begin{bmatrix} 0.3714 & -0.3714 & -0.2981 \\ -0.5571 & -0.5571 & 0.7454 \\ 0.7428 & 0.7428 & 0.5963 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}$$

Solving in Matlab yields:

$$\boxed{\begin{aligned} T_1 &= 14.28 \text{ N} \\ T_2 &= 5.86 \text{ N} \\ T_3 &= 14.52 \text{ N} \end{aligned}}$$

Alternately, see Matlab code (modified from problem 10.17) on previous page.

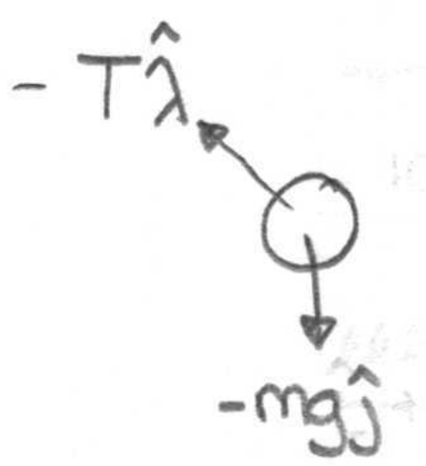
10.26

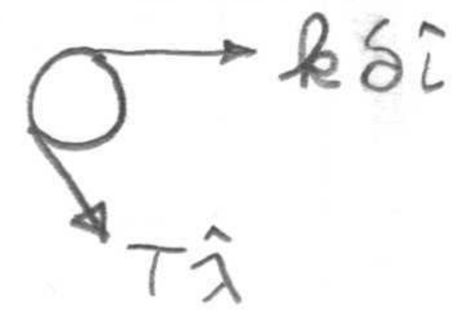


Given: $k = 200$ N/m
$l_0 = 2$ m
$l_{AP} = 8\text{m} = l_r$
$m = 100$ kg
Assume no air drag.

a) $\vec{r} = x\hat{i} + y\hat{j}$, $\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$, Find $\vec{a}$

PERSON: (1) PULLEY: (2)





$$\hat{\lambda} = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{i} + y\hat{j}}{\sqrt{x^2+y^2}}$$

δ = extension of spring = $l - l_0$, where $l = 10 - (l_r - \sqrt{x^2+y^2})$

$$\therefore l = 10 - 8 + \sqrt{x^2+y^2} = 2 + \sqrt{x^2+y^2}$$

$$\therefore \delta = l - l_0 = \sqrt{x^2+y^2}$$

From FBD 2: $T = k\delta$ (tension on both sides of pulley must be equal) $\therefore T = k\sqrt{x^2+y^2}$

From FBD 1: $\sum \vec{F} = m\vec{a} = -T\hat{\lambda} - mg\hat{j}$

$$\therefore -k\sqrt{x^2+y^2}\,\frac{x\hat{i} + y\hat{j}}{\sqrt{x^2+y^2}} - mg\hat{j} = m\vec{a}$$

OR $-kx\hat{i} - (ky+mg)\hat{j} = m\vec{a}$ - plug in values

$$\vec{a} = \frac{-200\,\text{N/m}\,x}{100\,\text{kg}}\hat{i} - \frac{200\,\text{N/m}\,y + 100\,\text{kg}\,(10\,\text{m/s}^2)}{100\,\text{kg}}\hat{j} = \boxed{-2x\,[s^{-2}]\hat{i} - (2y + 10\,\text{m})[s^{-2}]\hat{j}}$$

Problem 10.26 (b).

```
function Prob1026()
% Problem 10.26 Solution
% March 11, 2008

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  VARIABLES (Assume consistent units)
%    r  = displacement (vector with x and y components)
%    v  = dr/dt

% INTIAL CONDITIONS
r0= [1 -5]'; % initial position
v0= [0 0]'; % initial velocity
z0= [r0;v0]; % pack variables

tspan =[0 pi/sqrt(2)]; %time interval of integration

[t zarray] = ode45(@rhs,tspan, z0);

% Unpack Variables
r= zarray(:,1:2);

disp(r(end,:));

% ANSWER:
% ans =
%
%    -1.0000    -5.0000   (meters)
%

end

% THE DIFFERENTIAL EQUATION 'The Right Hand Side'
function zdot = rhs(t,z)

%Unpack variables
r= z(1:2);
v= z(3:4);

%The equations
rdot= v;
vdot= [-2*r(1) -2*r(2)-10]';

% Pack the rate of change of r and v
zdot= [rdot; vdot];

end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

b) See Matlab code on previous page

c) From part (a),

$$\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} = -2x\hat{\imath} - (2y+10)\hat{\jmath}$$

$$\therefore \quad (\ddot{x} = -2x)\hat{\imath}\cdot\hat{\imath} \rightarrow \ddot{x} = -2x \quad (1)$$

$$(\ddot{y} = -2y - 10)\hat{\jmath}\cdot\hat{\jmath} \rightarrow \ddot{y} = -2y - 10 \quad (2)$$

Solve (1) and (2) with $\vec{r}(0) = \hat{\imath} - 5\hat{\jmath}$, $\vec{v}(0) = \vec{0}$

(1) $\ddot{x} = -2x$, so $x(t) = A\sin(\sqrt{2}t) + B\cos(\sqrt{2}t)$

$\dot{x}(t) = \sqrt{2}A\cos(\sqrt{2}t) - \sqrt{2}B\sin(\sqrt{2}t)$

$\dot{x}(0) = 0 = \sqrt{2}A \quad \therefore \quad A = 0$

$x(0) = 1 = B\cos(0) \quad \therefore \quad B = 1$

$\therefore \quad x(t) = \cos(\sqrt{2}t)$

(2) $\ddot{y} = -2y - 10$, so $y(t) = C\sin(\sqrt{2}t) + D\cos(\sqrt{2}t) - \overset{y_P}{5}$

$\dot{y}(0) = 0 = \sqrt{2}C \quad \therefore \quad C = 0$

$y(0) = -5 = D\cos(0) - 5 \quad \therefore \quad D = 0$

$\therefore \quad y(t) = -5$

$$\vec{r}(t) = \cos\sqrt{2}t\,\hat{\imath} - 5\hat{\jmath} = x(t)\hat{\imath} + y(t)\hat{\jmath}$$

$$\therefore \quad \vec{r}\left(\tfrac{\pi}{\sqrt{2}}\right) = \cos\left(\sqrt{2}\tfrac{\pi}{\sqrt{2}}\right)\hat{\imath} - 5\hat{\jmath} = \boxed{-\hat{\imath} - 5\hat{\jmath}\ [\mathrm{m}]}$$

10.32

Given: $m = 1\,\text{kg}$, $\theta = 30°$, $v_0 = 172\ \text{m/s}$

$F_d = cv^2$, $c = 0.01\ \text{kg/m}$, $g = 10\ \text{m/s}^2$

a) See attached code (p. 9)

b) Assume 1D motion in the direction of $\vec{v}$ (gravity is negligible)

FBD:

$-c|\vec{v}|\vec{v}$

$$-c|\vec{v}|\vec{v} = m\vec{a}$$

OR

$$-cv^2 = ma$$

We have $m\dot{v} = -cv^2$, so $\dfrac{dv}{v^2} = \dfrac{-c}{m}dt$

$$\therefore \quad \frac{-1}{v} = \frac{-c}{m}t + c_1 \qquad v(0) = v_0$$

$$\frac{-1}{v_0} = c_1 \quad \rightarrow \quad \frac{-1}{v} = \frac{-c}{m}t - \frac{1}{v_0}$$

$$\text{OR} \quad \frac{-1}{v} = \frac{-ctv_0 - m}{mv_0} \quad \rightarrow \quad v = \frac{mv_0}{m + ctv_0}$$

$$r(t) = \int v(t)\,dt = \frac{mv_0}{c\cancel{v_0}}\ln(m + ctv_0) + c_2$$

$$r(0) = 0 = \ln(m) + c_2 \quad \therefore \quad c_2 = -\ln m$$

$$r(t) = \frac{m}{c}\ln(m + ctv_0) - \ln(m)$$

$$r(1) = \frac{1\,\text{kg}}{.01\ \text{kg/m}}\ln\left(1\,\text{kg} + 0.01\ \text{kg/m}\,(1\,\text{s})(172\ \text{m/s})\right) - \ln(1\,\text{kg})$$

$$= 100\ \text{m}$$

$$\therefore \quad y(1\,\text{s}) = r\sin 30° = \boxed{50\,\text{m}}$$

Problem 10.32

```
function Prob1032()
% Problem 10.32 Solution
% March 11, 2008

%  VARIABLES (Assume consistent units)
%   r  = displacement (vector with x and y components)
%   v  = dr/dt

% Define Variables
theta= 30; % degrees
v0= 172; % m/s
g= 10; % m/s^2
m= 1; % kg
c= .01; % kg/m

% INTIAL CONDITIONS
r0= [0 0]'; % initial position
v0= [v0*cosd(theta) v0*sind(theta)]'; % initial velocity
z0= [r0;v0]; % pack variables

tspan =linspace(0,1); %time interval of integration

[t zarray] = ode45(@rhs,tspan, z0, [], m, c, g);

% Unpack Variables
r= zarray(:,1:2);

disp(r(end,:));

% ANSWER:
% ans =
%
%    87.0044    46.5011   (meters)
%

end

% THE DIFFERENTIAL EQUATION 'The Right Hand Side'
function zdot = rhs(t, z, m, c, g)

%Unpack variables
r= z(1:2);
v= z(3:4);

%The equations
rdot= v;
vdot= -norm(v)*c/m*v - [0;g];

% Pack the rate of change of r and v
zdot= [rdot; vdot];

end
```